MATH 135 — QUIZ 12 SOLUTIONS — JAMES HOLLAND
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Question 1. Calculate the following definite integrals:

i. $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x \,\mathrm{d}x$; and

ii. $\displaystyle\int_{-1}^{2} (3x+4)^{100} \,\mathrm{d}x$.

Solution .:.

i. The indefinite integral $\int \cos x \,\mathrm{d}x = \sin x + c$. Therefore

$$\int_{-\pi/2}^{\pi/2} \cos x \,\mathrm{d}x = \sin x\Big]_{-\pi/2}^{\pi/2} = \sin(\pi/2) - \sin(-\pi/2) = 1 - (-1) = 2.$$

ii. Using a u-substitution of $u = 3x+4$ yields

$$\int (3x+4)^{100} \,\mathrm{d}x = \int u^{100}\frac{1}{3} \,\mathrm{d}u = \frac{1}{303}u^{101} + c = \frac{(3x+4)^{101}}{303} + c.$$

Therefore the definite integral can be calculated by

$$\int_{-1}^{2} (3x+4)^{100} \,\mathrm{d}x = \frac{(3x+4)^{101}}{303}\Bigg]_{-1}^{2} = \frac{10^{101}-1}{303},$$

which can also be written as one hundred '9's over "303", or one hundred '3's over "101".

Question 2. Calculate the following derivatives: (*Hint*: don't try to calculate $\int e^{x^2} \,\mathrm{d}x$)

i. $\displaystyle\frac{\mathrm{d}}{\mathrm{d}t}\int_{3}^{t} e^{x^2} \,\mathrm{d}x$; and

ii. $\displaystyle\frac{\mathrm{d}}{\mathrm{d}t}\int_{3}^{2t+1} e^{x^2} \,\mathrm{d}x$.

Solution .:.

i. By the fundamental theorem of calculus, this is just e^{t^2}.

ii. Using part (i), if $F(t) = \int_3^t e^{x^2} \,\mathrm{d}x$ has a derivative $F'(t) = e^{t^2}$, then $\int_3^{2t+1} e^{x^2} \,\mathrm{d}x = F(2t+1)$ has, by the chain rule, a derivative of

$$\frac{\mathrm{d}}{\mathrm{d}t}F(2t+1) = F'(2t+1)\cdot\frac{\mathrm{d}}{\mathrm{d}t}(2t+1) = e^{(2t+1)^2}\cdot 2.$$